Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 1

Tuesday 12 May 2015 (morning)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **Mathematical studies SL formula booklet** is required for this paper.
- Answer all questions.
- Write your answers in the boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

16 pages

16EP01

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Write your answers in the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. $T = \dfrac{(\tan(2z)+1)(2\cos(z)-1)}{y^2 - x^2}$, where $x = 9$, $y = 41$ and $z = 30°$.

(a) Calculate the **exact** value of T. [2]

(b) Give your answer to T correct to

    (i) two significant figures;

    (ii) three decimal places. [2]

Pyotr estimates the value of T to be 0.002.

(c) Calculate the percentage error in Pyotr's estimate. [2]

Working:

Answers:

(a) . . . .

(b) (i) . . . .

    (ii) . . . .

(c) . . . .

16EP02

2.   The IB grades attained by a group of students are listed as follows.

$$6 \quad 4 \quad 5 \quad 3 \quad 7 \quad 3 \quad 5 \quad 4 \quad 2 \quad 5$$

(a)   Find the median grade.   [2]

(b)   Calculate the interquartile range.   [2]

(c)   Find the probability that a student chosen at random from the group scored
at least a grade 4.   [2]
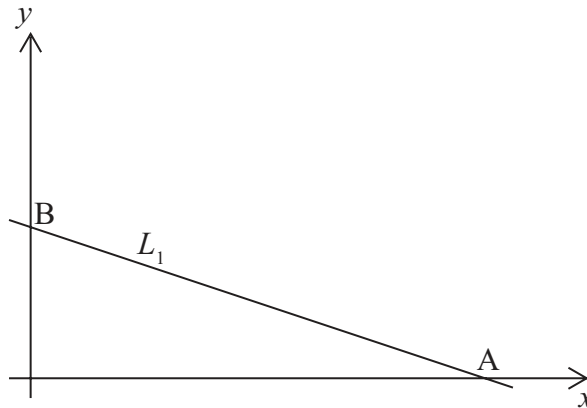
Working:

Answers:

(a)   . . . .

(b)   . . . .

(c)   . . . .

3.   The diagram shows the straight line L_1, which intersects the x-axis at $A(6, 0)$ and the y-axis at $B(0, 2)$.

(a)   Write down the coordinates of M, the midpoint of line segment AB.   [2]

(b)   Calculate the gradient of L_1.   [2]

The line L_2 is parallel to L_1 and passes through the point $(3, 2)$.

(c)   Find the equation of L_2. Give your answer in the form $y = mx + c$.   [2]

Working:

Answers:

(a)   . . . .

(b)   . . . .

(c)   . . . .

4.   Identical mosquito traps are placed at different distances from a lake. On one day the number of mosquitoes caught in 10 of the traps is recorded.

Distance, m (x)	8	15	22	30	34	45	50	60	74	82
Number of mosquitoes (y)	78	75	72	67	66	59	59	53	48	43

It is believed the number of mosquitoes caught varies linearly with the distance, in metres, of the trap from the lake.

(a)   Find

    (i)   Pearson's product–moment correlation coefficient, r;

    (ii)   the equation of the regression line y on x.   [4]

(b)   Use the equation of the regression line y on x to estimate the number of mosquitoes caught in a trap that is $28\,\text{m}$ from the lake.   [2]

Working:

Answers:

(a)   (i)   . . . .

    (ii)   . . . .

       . . . .

(b)   . . . .

Turn over

5. Assume the Earth is a perfect sphere with radius $6371\,\text{km}$.

    (a)   Calculate the volume of the Earth in km^3. Give your answer in the form $a \times 10^k$,
           where $1 \leq a < 10$ and $k \in \mathbb{Z}$.   [3]

The volume of the Moon is $2.1958 \times 10^{10}\,\text{km}^3$.

    (b)   Calculate how many times greater in volume the Earth is compared to the Moon.
           Give your answer correct to the nearest **integer**.   [3]

Working:

Answers:

(a)   . . . .

(b)   . . . .

16EP06

6. Pietro arrives in Singapore and, at the airport, changes 800 euros (EUR) to Singapore dollars (SGD).

The bank rates quoted at the airport for exchanging EUR with SGD are given in the following table. Also given are the rates for exchanging SGD with British pounds (GBP) and US dollars (USD). There is no commission charged on exchanges.

Bank Buys	Bank Sells
1 EUR = 1.55 SGD	1 EUR = 1.75 SGD
1 GBP = 1.92 SGD	1 GBP = 2.05 SGD
1 USD = 1.15 SGD	1 USD = 1.28 SGD

(a)   Calculate the number of SGD Pietro receives.   [2]

Pietro also has 100 GBP that he wishes to change to USD for a trip to Cambodia.
To perform this transaction, the GBP must first be converted to SGD and then to USD.

(b)   Calculate the number of USD Pietro receives.   [4]

Working:

Answers:

(a) . . . .

(b) . . . .

7.   The second term of an arithmetic sequence is 30. The fifth term is 90.

   (a)   Calculate

      (i)   the common difference of the sequence;

      (ii)   the first term of the sequence.   [3]

   The first, second and fifth terms of this arithmetic sequence are the first three terms of a geometric sequence.

   (b)   Calculate the seventh term of the **geometric** sequence.   [3]

Working:

Answers:

(a)   (i)   . . . .

      (ii)   . . . .

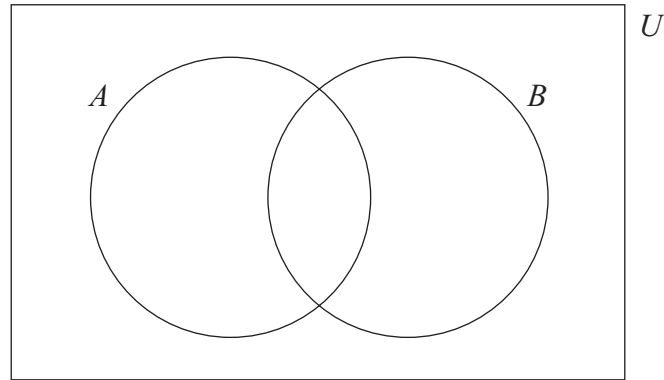(b)   . . . .

16EP08

8.   Aleph has an unbiased cubical (six faced) die on which are written the numbers
1, 2, 3, 4, 5 and 6.
Beth has an unbiased tetrahedral (four faced) die on which are written the numbers
2, 3, 5 and 7.

(a)   Complete the Venn diagram with the numbers written on Aleph's die (A) and
Beth's die (B).

[2]

(b)   Find $n(B \cap A')$.   [2]

Aleph and Beth are each going to roll their die once only. Shin says the probability that each
die will show the same number is $\dfrac{1}{8}$.

(c)   Determine whether Shin is correct. Give a reason.   [2]

Working:

Answers:

(b)   . . . .

(c)   . . . .

. . . .

. . . .

9.   A right pyramid has apex V and rectangular base $ABCD$, with $AB = 8$ cm, $BC = 6$ cm and $VA = 13$ cm. The vertical height of the pyramid is VM.

diagram not to scale

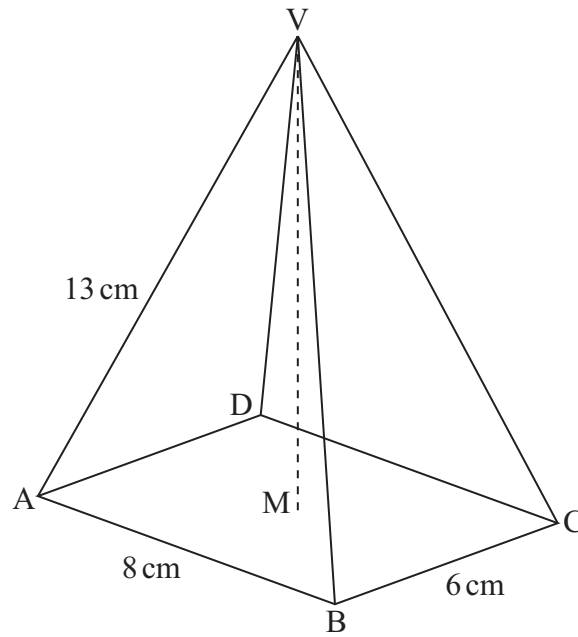(a)   Calculate VM.   [4]

(b)   Calculate the volume of the pyramid.   [2]

Working:

Answers:

(a)   . . . .

(b)   . . . .

10. Pierre invests 5000 euros in a fixed deposit that pays a nominal annual interest rate of 4.5%, compounded **monthly**, for seven years.

(a) Calculate the value of Pierre's investment at the end of this time. Give your answer correct to two decimal places. [3]

Carla has 7000 dollars to invest in a fixed deposit which is compounded **annually**.
She aims to double her money after 10 years.

(b) Calculate the minimum annual interest rate needed for Carla to achieve her aim. [3]

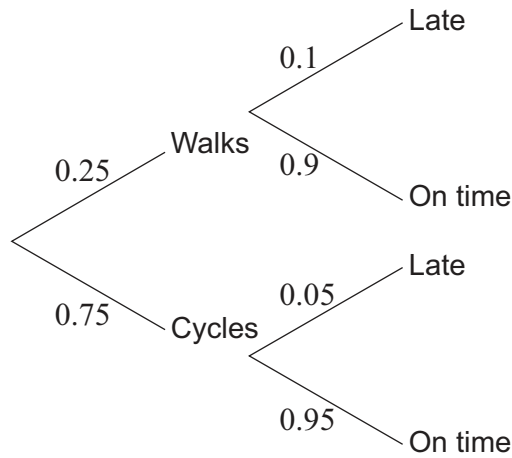**Working:**

Answers:

(a) . . . .

(b) . . . .

Turn over

11. Peter either walks or cycles to work. The probability that he walks is 0.25. If Peter walks to work, the probability that he is late is 0.1. If he cycles to work, the probability that he is late is 0.05. The tree diagram for this information is shown.

On a day chosen at random, Peter walked to work.

(a) Write down the probability that he was on time. [1]

For a different day, also chosen at random,

(b) find the probability that Peter cycled to work and was late; [2]

(c) find the probability that, given Peter was late, he cycled to work. [3]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

12. An iron bar is heated. Its length, L, in millimetres can be modelled by a linear function, $L = mT + c$, where T is the temperature measured in degrees Celsius $(^\circ C)$.

At $150\,^\circ C$ the length of the iron bar is $180\,\text{mm}$.

(a) Write down an equation that shows this information. [1]

At $210\,^\circ C$ the length of the iron bar is $181.5\,\text{mm}$.

(b) Write down an equation that shows this second piece of information. [1]

(c) Hence, find the length of the iron bar at $40\,^\circ C$. [4]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

Turn over

13. The weight, W, of bags of rice follows a normal distribution with mean $1000\,\text{g}$ and standard deviation $4\,\text{g}$.

(a) Find the probability that a bag of rice chosen at random weighs between $990\,\text{g}$ and $1004\,\text{g}$. [2]

95% of the bags of rice weigh less than k grams.

(b) Find the value of k. [2]

For a bag of rice chosen at random, $\text{P}(1000 - a < W < 1000 + a) = 0.9$.

(c) Find the value of a. [2]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

14. Consider the quadratic function, $f(x) = px(q - x)$, where p and q are positive integers.
The graph of $y = f(x)$ passes through the point $(6, 0)$.

(a) Calculate the value of q. [2]

The vertex of the function is $(3, 27)$.

(b) Find the value of p. [2]

(c) Write down the range of f. [2]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

15. A cuboid has a rectangular base of width $x\,\text{cm}$ and length $2x\,\text{cm}$. The height of the cuboid is $h\,\text{cm}$. The total length of the edges of the cuboid is $72\,\text{cm}$.

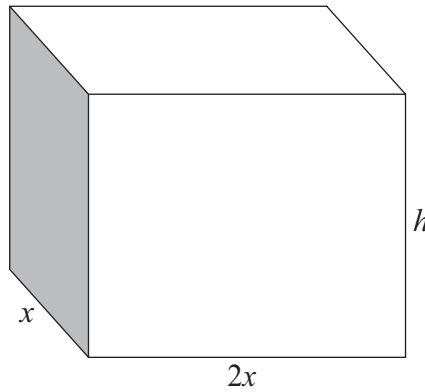**diagram not to scale**

The volume, V, of the cuboid can be expressed as $V = ax^2 - 6x^3$.

(a)   Find the value of a.   [3]

(b)   Find the value of x that makes the volume a maximum.   [3]

Working:

Answers:

(a)   . . . .

(b)   . . . .